

**Problem 16)** The binomial expansion of  $(x + 1)^n$  is given by

$$(x + 1)^n = \sum_{k=0}^n \binom{n}{k} x^k.$$

Differentiation with respect to  $x$  yields

$$n(x + 1)^{n-1} = \sum_{k=0}^n k \binom{n}{k} x^{k-1}.$$

Setting  $x = 1$ , and noting that the term corresponding to  $k = 0$  is zero, we find

$$\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}.$$

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